

On Separation Axioms in Soft Minimal Spaces

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ABSTRACT: The aim of this paper is to introduce the concept of some separation axioms in soft minimal spaces such as Soft T_0 -space, Soft T_1 -space, Soft T_2 -space, soft regular, and soft normal space which involve only pair of distinct soft points and we study some of their properties.

KEYWORDS: Soft T_0 -space, Soft T_1 -space, Soft T_2 -space, Soft semi Hausdroff space, Soft Pseudo Hausdroff space, Soft Uryshon space, soft regular, soft normal, Completely soft normal space.

I. INTRODUCTION

V. Popa and T. Noiri [10] introduced the concept of minimal structure. Also they introduced the notion of m_X -open set and m_X -closed set and characterize those sets using m_X -closure and m_X -interior operators respectively. Further they Introduced M-continuous functions and studied some of its basic properties. Recently, many researchers improved minimal spaces see [1] [2] [3] and. M. Tunapan [13] introduced some separation axioms in biminimal structure spaces such as CT_0 -space, CT_1 -space, C-Hausdroff space, C-regular space and C-normal space. D. Molodtsov [9] introduced the concept of soft set theory as a general mathematical tool to deal with uncertainties while modeling the problems with incomplete information. Many researches improved the concept of soft sets. Shabir and Naz [12] initiated the study of soft topological spaces. They defined soft topology as a collection of soft sets over X . Consequently, they defined basic notions of soft topological spaces such as soft open and soft closed sets, soft subspace, soft closure, soft neighborhood of a point and soft separation axioms are also introduced and their basic properties are investigated by them. In this paper, we introduce and study some separation axioms in soft minimal spaces.

II. PRELIMINARIES

2.1. Definition. [9]. Let U be an initial universe and E is a set of parameters. Let $P(U)$ denotes the power set of U and A is a nonempty subset of E . A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε - approximate elements of the soft set (F, A) .

2.2. Definition. [7]. Let X be an initial universe set, E be the set of parameters and $A \subseteq E$. Let F_A be a nonempty soft set over X and $\tilde{p}(F_A)$ is the soft power set of F_A . A subfamily \tilde{m} of $\tilde{p}(F_A)$ is called a soft minimal set over X if $F_\phi \in \tilde{m}$ and $F_A \in \tilde{m}$.

(F_A, \tilde{m}) or (X, \tilde{m}, E) is called a soft minimal space over X . Each member of \tilde{m} is said to be \tilde{m} -soft open (or soft \tilde{m} -open) set and the complement of an \tilde{m} -soft open (or soft \tilde{m} -open) set is said to be \tilde{m} -soft closed (or soft \tilde{m} -closed) set over X .

2.3. Definition. [7]. Let (F_A, \tilde{m}) be a soft minimal space over X . For a soft subset F_B of F_A , the \tilde{m} -soft closure (or soft \tilde{m} -closure) of F_B and \tilde{m} -soft interior (or soft \tilde{m} -interior) of F_B are defined as follows:

- (1) $\tilde{m}Cl(F_B) = \cap \{F_\alpha : F_B \subseteq F_\alpha, F_\alpha \in \tilde{m}\}$,
- (2) $\tilde{m}Int(F_B) = \cup \{F_\beta : F_\beta \subseteq F_B, F_\beta \in \tilde{m}\}$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

2.4. Definition: [7]. Let (F_A, \tilde{m}) be a soft minimal space and F_Y be a soft subset of F_A . Define soft minimal set \tilde{m}_{F_Y} on F_Y as follows: $\tilde{m}_{F_Y} = \{F_B \cap F_Y \mid F_B \in \tilde{m}\}$. Then (F_Y, \tilde{m}_{F_Y}) is called a soft minimal subspace of (F_A, \tilde{m}) .

III. SOFT T_0 - SPACE, SOFT T_1 - SPACE and SOFT T_2 - SPACE in SOFT MINIMAL SPACES

3.1. Definition: Let (F_A, \tilde{m}) be a soft minimal space over X and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. If there exist at least one soft \tilde{m} -open set U_B or V_B such that $(y, u) \notin U_B$ or $(x, u) \notin V_B$, then (F_A, \tilde{m}) is called a soft T_0 -space.

3.2. Definition: Let (F_A, \tilde{m}) be a soft minimal space over X and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. If there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B, (y, u) \notin U_B$ and $(y, u) \in V_B, (x, u) \notin V_B$ then (F_A, \tilde{m}) is called a soft T_1 -space.

3.3. Definition: Let (F_A, \tilde{m}) be a soft minimal space over X and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. If there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B, (y, u) \in V_B$, and $U_B \cap V_B = F_\phi$, then (F_A, \tilde{m}) is called a soft T_2 -space.

3.4. Example: Let $U = \{u_1, u_2\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. Then $F_{A1} = \{(x_1\{u_1\})\}, F_{A2} = \{(x_1\{u_2\})\}, F_{A3} = \{(x_1\{u_1, u_2\})\}, F_{A4} = \{(x_2\{u_1\})\}, F_{A5} = \{(x_2\{u_2\})\}, F_{A6} = \{(x_2\{u_1, u_2\})\}, F_{A7} = \{(x_1\{u_1\}), (x_2\{u_1\})\}, F_{A8} = \{(x_1\{u_1\}), (x_2\{u_2\})\}, F_{A9} = \{(x_1\{u_1\}), (x_2\{u_1, u_2\})\}, F_{A10} = \{(x_1\{u_2\}), (x_2\{u_1\})\}, F_{A11} = \{(x_1\{u_2\}), (x_2\{u_2\})\}, F_{A12} = \{(x_1\{u_2\}), (x_2\{u_1, u_2\})\}, F_{A13} = \{(x_1\{u_1, u_2\}), (x_2\{u_1\})\}, F_{A14} = \{(x_1\{u_1, u_2\}), (x_2\{u_2\})\}, $F_{A15} = F_A, F_{A16} = F_\phi$ are all soft subset of $F_A, |P(F_A)| = 2^4 = 16$.$

Take $\tilde{m} = \{F_\phi, F_{A2}, F_{A5}, F_{A8}, F_{A10}, F_A\}$, then (F_A, \tilde{m}) is soft T_2 -space.

3.5. Proposition:

- (1) Every soft T_1 -space is soft T_0 -space.
- (2) Every soft T_2 -space is soft T_1 -space.

Proof: Let (F_A, \tilde{m}) be a soft minimal space over X and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$.

(1) Obvious

(2) Let (F_A, \tilde{m}) is a soft T_2 -space, then by definition of soft T_2 -space $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, if there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B, (y, u) \in V_B$ and $U_B \cap V_B = F_\phi$. Since $U_B \cap V_B = F_\phi$, So $(x, u) \notin V_B$ and $(y, u) \notin U_B$. Thus it follows (F_A, \tilde{m}) is a soft T_1 -space.

Note that every soft T_1 -space is soft T_0 -space. Every soft T_2 -space is soft T_1 -space. But the converses do not hold in general.

3.6. Example: Let (F_A, \tilde{m}) be a soft minimal space where Let $U = \{u_1, u_2\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. $\tilde{m} = \{F_\phi, F_{A7}, F_A\}$. Thus (F_A, \tilde{m}) is soft T_0 -space but not soft T_1 -space.

3.7. Example: Let (F_A, \tilde{m}) be a soft minimal space where Let $U = \{u_1, u_2\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. $\tilde{m} = \{F_\phi, F_{A2}, F_{A9}, F_{A11}, F_{A13}, F_A\}$, Thus (F_A, \tilde{m}) is soft T_1 -space but not soft T_2 -space.

3.8. Lemma: Let (F_A, \tilde{m}) be a soft minimal space and let (F_Y, \tilde{m}_{F_Y}) be a soft closed subspace of (F_A, \tilde{m}) . If U_B is soft \tilde{m} -open subset of (F_A, \tilde{m}) , then $U_B \cap F_Y$ is also soft \tilde{m}_{F_Y} -open subset of (F_Y, \tilde{m}_{F_Y}) .

Proof: Let U_B be a soft \tilde{m} -open subset of (F_A, \tilde{m}) . Since (F_Y, \tilde{m}_{F_Y}) be a soft closed subset of (F_A, \tilde{m}) . Then $F_A - U_B$ is a soft \tilde{m} -closed subset of (F_A, \tilde{m}) . Since (F_Y, \tilde{m}_{F_Y}) be a soft closed subset (F_A, \tilde{m}) .

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 6, Issue 7, July 2017

But $(F_A - U_B) \cap F_Y = F_Y - (U_B \cap F_Y)$. Therefore, $F_Y - (U_B \cap F_Y)$ is a soft \tilde{m}_{F_Y} -closed subset of (F_Y, \tilde{m}_{F_Y}) . Hence, $(U_B \cap F_Y)$ is a soft \tilde{m}_{F_Y} -open subset of (F_Y, \tilde{m}_{F_Y}) .

3.9. Proposition: Let (F_Y, \tilde{m}_{F_Y}) be a soft minimal subspace of (F_A, \tilde{m}) . If (F_A, \tilde{m}) is soft T_0 -space then (F_Y, \tilde{m}_{F_Y}) is also a soft T_0 -space.

Proof: Let (F_A, \tilde{m}) is a soft minimal T_0 -space and (F_Y, \tilde{m}_{F_Y}) be the soft subspace of (F_A, \tilde{m}) . If $(x, u), (y, u) \in F_Y$ such that $(x, u) \neq (y, u)$. Since $(F_Y, \tilde{m}_{F_Y}) \subseteq (F_A, \tilde{m})$, then there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \notin U_B$ or $(y, u) \in V_B$, $(x, u) \notin V_B$. Then, $(x, u) \in U_B \cap F_Y$ and $(y, u) \notin U_B \cap F_Y$. Similarly, it can be proved that $(y, u) \in V_B \cap F_Y$ and $(x, u) \notin V_B \cap F_Y$. Thus (F_Y, \tilde{m}_{F_Y}) is soft T_0 -space.

3.10. Proposition: Let (F_Y, \tilde{m}_{F_Y}) be a soft minimal subspace of (F_A, \tilde{m}) . If (F_A, \tilde{m}) is soft T_1 -space, then (F_Y, \tilde{m}_{F_Y}) is also a soft T_1 -space.

Proof: The proof is similar to the proof of proposition 3.9.

3.11. Proposition: Let (F_Y, \tilde{m}_{F_Y}) be a soft minimal subspace of (F_A, \tilde{m}) . If (F_A, \tilde{m}) is soft T_2 -space, then (F_Y, \tilde{m}_{F_Y}) is also a soft T_2 -space.

Proof: Let (F_A, \tilde{m}) is a soft minimal T_2 -space and (F_Y, \tilde{m}_{F_Y}) be the soft subspace of (F_A, \tilde{m}) . If $(x, u), (y, u) \in F_Y$ such that $(x, u) \neq (y, u)$. Since $(F_Y, \tilde{m}_{F_Y}) \subseteq (F_A, \tilde{m})$, then there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $U_B \cap V_B = F_\phi$. Therefore, $(x, u) \in U_B \cap F_Y$, $(y, u) \in V_B \cap F_Y$ and $(U_B \cap F_Y) \cap (V_B \cap F_Y) = F_\phi$. Thus (F_Y, \tilde{m}_{F_Y}) is soft T_2 -space.

3.12. Proposition: Let (F_A, \tilde{m}) be a soft minimal space over X and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. If there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$ and $(y, u) \in (U_B)^C$ or $(y, u) \in V_B$ and $(x, u) \in (V_B)^C$, then (F_A, \tilde{m}) is a soft T_0 -space.

Proof: Let $(y, u) \in (U_B)^C$ such that $(x, u) \neq (y, u)$ and U_B and V_B are soft \tilde{m} -open sets such that $(x, u) \in U_B$ and $(y, u) \in (U_B)^C$ or $(y, u) \in V_B$ and $(x, u) \in (V_B)^C$. If $(y, u) \in (U_B)^C$. This implies that $(y, u) \notin U_B$. Similarly, we can show that if $(x, u) \in (V_B)^C$ implies $(x, u) \notin V_B$. Hence (F_A, \tilde{m}) is soft T_0 -space.

3.13. Theorem: Let (F_A, \tilde{m}) be a soft minimal space over X and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. If there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \in (U_B)^C$ and $(y, u) \in V_B$, $(x, u) \in (V_B)^C$, then (F_A, \tilde{m}) is a soft T_1 -space.

Proof: The proof is similar to the proof of proposition 3.12.

3.14. Theorem: A soft minimal space (F_A, \tilde{m}) is a soft T_1 -space if and only if for each singleton set $\{(x, u)\}$ is soft \tilde{m} -closed set in F_A .

Proof: Suppose F_A is soft T_1 -space. Let $(x, u) \in F_A$. Let $\{(y, u)\} \in \{(x, u)\}^C$, then $(x, u) \neq (y, u)$ and so there exist soft \tilde{m} -open set U_B containing (y, u) but not (x, u) and soft \tilde{m} -open set V_B containing (x, u) but not (y, u) , $(y, u) \in U_B \subseteq \{(x, u)\}^C$. Therefore $\{(x, u)\}^C$ is soft \tilde{m} -open set then $\{(x, u)\}$ is soft \tilde{m} -closed set.

Conversely, suppose $(x, u), (y, u) \in F_A$, such that $(x, u) \neq (y, u)$. Since $\{(x, u)\}$ is soft \tilde{m} -closed set, then $\{(x, u)\}^C$ is soft \tilde{m} -open set containing (y, u) but not (x, u) . Similarly, $\{(y, u)\}^C$ is soft \tilde{m} -open set containing (x, u) but not (y, u) . Therefore (F_A, \tilde{m}) is soft T_1 -space.

3.15. Theorem: A soft minimal space (F_A, \tilde{m}) is a soft T_2 -space and for any $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, then there exist soft \tilde{m} -closed sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \notin U_B$ and $(y, u) \in V_B$, $(x, u) \notin V_B$ and $U_B \cup V_B = F_A$.

Proof: Since (F_A, \tilde{m}) is a soft T_2 -space and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, there exist soft \tilde{m} -open sets K_B and L_B such that $(x, u) \in K_B$, $(y, u) \in L_B$ and $K_B \cap L_B = F_\phi$. Clearly $K_B \subseteq (L_B)^C$ and $L_B \subseteq (K_B)^C$. Hence, $(x, u) \in (L_B)^C$. Put $(L_B)^C = U_B$. This gives $(x, u) \in U_B$ and $(y, u) \notin U_B$. Also $(y, u) \in (K_B)^C$. Put $(K_B)^C = V_B$. This gives $(x, u) \notin V_B$ and $(y, u) \in V_B$. Therefore, $(x, u) \in U_B$ and $(y, u) \in V_B$. Hence, $U_B \cup V_B = (L_B)^C \cup (K_B)^C = F_A$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

3.16. Definition: A soft minimal space (F_A, \tilde{m}) is said to be soft semi Hausdroff if for every $(x, u) \neq (y, u) \in F_A$, there exist soft \tilde{m} -open set U_B or V_B such that $(x, u) \in U_B$ and $(y, u) \notin \tilde{m}Cl(U_B)$ or $(y, u) \in V_B$ and $(x, u) \notin \tilde{m}Cl(V_B)$.

3.17. Definition: A soft minimal space (F_A, \tilde{m}) is said to be soft Pseudo Hausdroff space if for every $(x, u) \neq (y, u)$ in F_A , there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \notin \tilde{m}Cl(U_B)$ and $(y, u) \in V_B$, $(x, u) \notin \tilde{m}Cl(V_B)$.

3.18. Theorem: A soft minimal space (F_A, \tilde{m}) is soft Pseudo Hausdroff, and then every soft subspace (F_Y, \tilde{m}_{F_Y}) of (F_A, \tilde{m}) is also soft pseudo Hausdroff.

Proof: Let soft minimal space (F_A, \tilde{m}) is a soft pseudo Hausdroff. Let (F_Y, \tilde{m}_{F_Y}) be the soft subspace of (F_A, \tilde{m}) . Since (F_A, \tilde{m}) is a soft pseudo Hausdroff, then for every $(x, u) \neq (y, u)$, there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \notin \tilde{m}Cl(U_B)$ and $(y, u) \in V_B$, $(x, u) \notin \tilde{m}Cl(V_B)$. Then $U_B \cap F_Y$ and $V_B \cap F_Y$ are soft \tilde{m}_{F_Y} -open sets in (F_Y, \tilde{m}_{F_Y}) such that $(x, u) \in U_B \cap F_Y$ and $(y, u) \notin \tilde{m}_{F_Y}Cl(U_B \cap F_Y)$ also $(y, u) \in V_B \cap F_Y$ and $(x, u) \notin \tilde{m}_{F_Y}Cl(V_B \cap F_Y)$. Therefore (F_Y, \tilde{m}_{F_Y}) is soft Pseudo Hausdroff.

3.19. Definition: A soft minimal space (F_A, \tilde{m}) is said to be soft Uryshon space if given $(x, u) \neq (y, u)$ in F_A , there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $\tilde{m}Cl(U_B) \cap \tilde{m}Cl(V_B) = F_\Phi$.

3.20. Theorem: In a soft minimal space (F_A, \tilde{m}) is soft Uryshon, then every soft subspace of a soft Uryshon space is soft Uryshon.

Proof: Let (F_A, \tilde{m}) is soft Uryshon space and let (F_Y, \tilde{m}_{F_Y}) be a soft subspace of (F_A, \tilde{m}) . Since (F_A, \tilde{m}) is soft Uryshon, then for every $(x, u) \neq (y, u)$, there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $\tilde{m}Cl(U_B) \cap \tilde{m}Cl(V_B) = F_\Phi$. Now $U_B \cap F_Y$ and $V_B \cap F_Y$ are soft \tilde{m}_{F_Y} -open sets such that $(x, u) \in U_B \cap F_Y$ and $(y, u) \in V_B \cap F_Y$ in (F_Y, \tilde{m}_{F_Y}) . Consider,

$$\begin{aligned} \tilde{m}_{F_Y}Cl(U_B \cap F_Y) \cap \tilde{m}_{F_Y}Cl(V_B \cap F_Y) &= [\tilde{m}_{F_Y}Cl(U_B \cap F_Y)] \cap [\tilde{m}_{F_Y}Cl(V_B \cap F_Y)] \\ &= [\tilde{m}_{F_Y}Cl(U_B) \cap \tilde{m}_{F_Y}Cl(V_B)] \cap F_Y \\ &= [\tilde{m}Cl(U_B) \cap \tilde{m}Cl(V_B)] \cap F_Y \\ &= F_\Phi \cap F_Y = F_\Phi. \end{aligned}$$

Hence (F_Y, \tilde{m}_{F_Y}) is soft Uryshon space.

IV. SOFT REGULAR T_3 -SPACE, SOFT NORMAL T_4 -SPACE IN SOFT MINIMAL SPACES

In this section, we introduce soft regular T_3 -space, soft normal T_4 -space and completely soft normal space.

4.1. Definition: Let (F_A, \tilde{m}) be soft minimal space. A soft minimal space (F_A, \tilde{m}) is said to be soft regular if for each soft point $(x, u) \in F_A$ and each soft \tilde{m} -closed set G_B not containing (x, u) , there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $G_B \not\supseteq V_B$ and $U_B \cap V_B = F_\Phi$.

4.2. Example: Let (F_A, \tilde{m}) be a soft minimal space where Let $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1\{u_1, u_2\}), (x_2\{u_1, u_2\})\}$. $\tilde{m} = \{F_\Phi, F_{A3}, F_{A6}, F_{A7}, F_{A11}, F_A\}$.

Take $G_B = F_{A6}$, $F_{A1} \notin G_B$, then there exists F_{A3} and F_{A6} are soft \tilde{m} -open sets such that $F_{A1} \in F_{A3}$, $F_{A6} \in F_{A6}$, $F_{A6} \not\supseteq F_{A6}$ and

$F_{A3} \cap F_{A6} = F_\Phi$ and similarly the other cases. Hence (F_A, \tilde{m}) is soft regular space.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

4.3. Proposition: Let (F_A, \tilde{m}) be a soft minimal space, G_B be soft \tilde{m} -closed set and $(x, u) \in F_A$ such that $(x, u) \notin G_B$. If (F_A, \tilde{m}) is soft regular space, then there exist soft \tilde{m} -open set U_B such that $(x, u) \in U_B$ and $U_B \cap G_B = F_\phi$.

Proof: It is obvious from Definition 4.1.

4.4. Proposition: Let (F_A, \tilde{m}) be a soft minimal space and $(x, u) \in F_A$. If (F_A, \tilde{m}) is soft regular space, then:

- (1) $(x, u) \in G_B$ if and only if $\{(x, u)\} \subseteq G_B$.
- (2) If $\{(x, u)\} \cap G_B = F_\phi$, then $(x, u) \notin G_B$.

Proof: It is obvious.

4.5. Theorem: Let (F_A, \tilde{m}) be a soft minimal space and $(x, u) \in F_A$. If (F_A, \tilde{m}) is soft regular space, then:

- (1) For a soft \tilde{m} -closed set G_B , $(x, u) \notin G_B$ if and only if $\{(x, u)\} \cap G_B = F_\phi$.
- (2) For a soft \tilde{m} -open set V_B , $(x, u) \notin V_B$ if and only if $\{(x, u)\} \cap V_B = F_\phi$.

Proof: (1) Suppose (F_A, \tilde{m}) be a soft regular space and $(x, u) \in F_A$. Let G_B be a soft \tilde{m} -closed set such that $(x, u) \notin G_B$. Then by proposition 4.3, there exist soft \tilde{m} -open set U_B such that $(x, u) \in U_B$ and $U_B \cap G_B = F_\phi$. It follows that $\{(x, u)\} \subseteq U_B$ from proposition 4.4 (1). Hence $\{(x, u)\} \cap G_B = F_\phi$. Conversely, if $\{(x, u)\} \cap G_B = F_\phi$, then $(x, u) \notin G_B$ from Proposition 4.4 (2).

(2) Let V_B be soft \tilde{m} -open set such that $(x, u) \notin V_B$. Assume $(x, u) \in V_B$. This means that, $\{(x, u)\} \cap V_B = F_\phi$. Hence $(V_B)^c$ is soft \tilde{m} -closed set such that $\{(x, u)\} \subseteq (V_B)^c$. It follows by (1) $\{(x, u)\} \cap (V_B)^c = F_\phi$. This implies that $\{(x, u)\} \subseteq V_B$ and so $(x, u) \in V_B$, which is contradiction with $(x, u) \notin V_B$. Therefore, $\{(x, u)\} \cap V_B = F_\phi$. Conversely, if $\{(x, u)\} \cap V_B = F_\phi$, then it is obvious that $(x, u) \notin V_B$. This completes the proof.

4.6. Corollary: Let (F_A, \tilde{m}) be a soft minimal space and $(x, u) \in F_A$. If (F_A, \tilde{m}) is soft regular space then the following are equivalent:

- (1) (F_A, \tilde{m}) is soft T_1 -space.
- (2) $(x, u) (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, there exist soft \tilde{m} -open sets U_B and V_B such that $\{(x, u)\} \subseteq U_B$ and $\{(y, u)\} \cap U_B = F_\phi$ and $(y, u) \subseteq V_B$ and $\{(x, u)\} \cap V_B = F_\phi$.

Proof: It is obvious from Theorem 4.5.

4.7. Theorem: Let (F_A, \tilde{m}) be a soft minimal space and $(x, u) \in F_A$. Then the following are equivalent:

- (1) (F_A, \tilde{m}) is soft regular space.
- (2) For every soft \tilde{m} -closed set G_B such that $\{(x, u)\} \cap G_B = F_\phi$, there exist soft \tilde{m} -open sets U_B and V_B such that $\{(x, u)\} \subseteq U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\phi$.

Proof: (1) \rightarrow (2)

Let G_B be a soft \tilde{m} -closed set such that $\{(x, u)\} \cap G_B = F_\phi$. Then $(x, u) \notin G_B$ from Theorem 4.5 (1). It follows by (1), there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\phi$. This means that, $\{(x, u)\} \subseteq U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\phi$.

(2) \rightarrow (1)

Let G_B be a soft \tilde{m} -closed set such that $(x, u) \notin G_B$. Then $\{(x, u)\} \cap G_B = F_\phi$ from Theorem 4.5(1). It follows by (2), there exist soft \tilde{m} -open sets U_B and V_B such that $\{(x, u)\} \subseteq U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\phi$. Hence $\{(x, u)\} \in U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\phi$. Thus, (F_A, \tilde{m}) is soft regular space.

4.8. Theorem: Let (F_A, \tilde{m}) be a soft minimal space, then (F_A, \tilde{m}) is soft regular if and only if for each soft \tilde{m} -open set U_B and $(x, u) \in U_B$, there exist soft \tilde{m} -open set V_B such that $(x, u) \in V_B$, $\tilde{m}Cl(V_B) \subseteq U_B$.

Proof: Let (F_A, \tilde{m}) be soft regular space. Let $(x, u) \in U_B$, where U_B is soft \tilde{m} -open. Let $H_B = (U_B)^c$, then H_B is soft \tilde{m} -closed, $(x, u) \notin H_B$. By the soft regularity of (F_A, \tilde{m}) , there exist soft \tilde{m} -open sets N_B and M_B such that $(x, u) \in M_B$, $H_B \subseteq N_B$ and $M_B \cap N_B = F_\phi$. Then $M_B \subseteq (N_B)^c$, $\tilde{m}Cl(M_B) \subseteq \tilde{m}Cl((N_B)^c) = (N_B)^c$. \rightarrow (1)

$H_B \subseteq (N_B)^c$, then $(N_B)^c \subseteq (H_B)^c = U_B$, then $(N_B)^c \subseteq U_B \rightarrow$ (2)

From (1) and (2), we have $(x, u) \in M_B$, $\tilde{m}Cl(M_B) \subseteq U_B$.

International Journal of Innovative Research in Science, Engineering and Technology

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Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

Conversely, let H_B be soft \tilde{m} -closed set and $(x, u) \notin H_B$. Let $U_B = (H_B)^C$, then U_B is soft \tilde{m} -open and $(x, u) \in U_B$. By hypothesis, there exist soft \tilde{m} -open set M_B such that $(x, u) \in M_B$, $\tilde{m}Cl(M_B) \subseteq U_B$, $H_B \subseteq [\tilde{m}Cl(M_B)]^C$. Since $(x, u) \in M_B$, $M_B \cap [\tilde{m}Cl(M_B)]^C = F_\Phi$. Hence (F_A, \tilde{m}) is soft regular.

4.9. Theorem: A soft minimal space (F_A, \tilde{m}) is soft regular space if and only if for each $(x, u) \in F_A$ and soft \tilde{m} -closed set G_B such that $(x, u) \notin G_B$, there exist soft \tilde{m} -open set U_B such that $(x, u) \in U_B$ and $\tilde{m}Cl(U_B) \cap G_B = F_\Phi$.

Proof: Suppose that (F_A, \tilde{m}) is a soft regular space. Let $(x, u) \in F_A$ and G_B be a soft \tilde{m} -closed such that $(x, u) \notin G_B$. Then there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. Now $U_B \cap V_B = F_\Phi$, $U_B \subseteq (V_B)^C$ and $G_B \subseteq V_B$, $(V_B)^C \subseteq (G_B)^C$. That implies $U_B \subseteq (V_B)^C \subseteq (G_B)^C$. This implies that $\tilde{m}Cl(U_B) \subseteq \tilde{m}Cl(V_B)^C = (V_B)^C \subseteq (G_B)^C$. Therefore $\tilde{m}Cl(U_B) \cap G_B = F_\Phi$.

Conversely, suppose $(x, u) \in F_A$ and G_B be soft \tilde{m} -closed set such that $(x, u) \notin G_B$. Then by hypothesis there exist a soft \tilde{m} -open set U_B such that $(x, u) \in U_B$ and $\tilde{m}Cl(U_B) \cap G_B = F_\Phi$. Now $\tilde{m}Cl(U_B) \cap G_B = F_\Phi$ implies that $G_B \subseteq [\tilde{m}Cl(U_B)]^C$. Also $U_B \subseteq \tilde{m}Cl(U_B)$ implies $[\tilde{m}Cl(U_B)]^C \subseteq (U_B)^C$. $U_B \subseteq [\tilde{m}Cl(U_B)]^C$. Therefore $U_B \cap [\tilde{m}Cl(U_B)]^C = F_\Phi$. Then there exist soft \tilde{m} -open sets U_B and $\tilde{m}Cl(U_B)^C$ such that $(x, u) \in U_B$, $G_B \subseteq [\tilde{m}Cl(U_B)]^C$ and $U_B \cap [\tilde{m}Cl(U_B)]^C = F_\Phi$. Hence (F_A, \tilde{m}) is soft regular space.

4.10. Definition: Let (F_A, \tilde{m}) be a soft minimal space. Then (F_A, \tilde{m}) is said to be a soft T_3 -space, if it is a soft regular and a soft T_1 -space

4.11. Theorem: Every soft T_3 -space is a soft T_2 -space.

Proof: Let (F_A, \tilde{m}) be a soft T_3 -space and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. Since (F_A, \tilde{m}) is a soft T_1 -space, and $\{(y, u)\}$ is soft \tilde{m} -closed and $(x, u) \notin \{(y, u)\}$. It follows from soft regularity, there exist soft \tilde{m} open sets U_B and V_B such that $(x, u) \in U_B$, $\{(y, u)\} \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. Thus $(x, u) \in U_B$, $(y, u) \in \{(y, u)\} \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. Therefore, (F_A, \tilde{m}) is soft T_2 -space.

4.12. Theorem: A soft subspace (F_Y, \tilde{m}_{F_Y}) of a soft T_3 -space (F_A, \tilde{m}) is soft T_3 -space.

Proof: First we want to prove that (F_Y, \tilde{m}_{F_Y}) is soft T_1 -space. If $(x, u), (y, u) \in F_Y$ such that $(x, u) \neq (y, u)$. Since $(F_Y, \tilde{m}_{F_Y}) \subseteq (F_A, \tilde{m})$, then there exist soft \tilde{m} -open sets U_B and V_B such that $(x, u) \in U_B$, $(y, u) \notin U_B$ and $(y, u) \in V_B$, $(x, u) \notin V_B$. Therefore, $(x, u) \in U_B \cap F_Y$ and $(y, u) \notin U_B \cap F_Y$ and $(y, u) \in V_B \cap F_Y$ and $(x, u) \notin V_B \cap F_Y$. Thus (F_Y, \tilde{m}_{F_Y}) is soft T_1 -space. Next we want to prove (F_Y, \tilde{m}_{F_Y}) is soft regular space. Let $(y, u) \in F_Y$ and H_B be soft \tilde{m}_{F_Y} -closed set in (F_Y, \tilde{m}_{F_Y}) such that $(y, u) \notin H_B$. Then $H_B = G_B \cap F_Y$, where G_B is soft \tilde{m} -closed set in (F_A, \tilde{m}) . Hence, $(y, u) \notin G_B \cap F_Y$. But $(y, u) \in F_Y$, so $(y, u) \notin G_B$. Since (F_A, \tilde{m}) is soft T_3 -space, so there exist soft \tilde{m} -open sets U_B and V_B in (F_A, \tilde{m}) such that $(y, u) \in U_B$, $G_B \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. Take $M_B = U_B \cap F_Y$ and $N_B = V_B \cap F_Y$, then $U_B \cap F_Y$ and $V_B \cap F_Y$ are soft \tilde{m}_{F_Y} -open sets in (F_Y, \tilde{m}_{F_Y}) such that $(y, u) \in M_B$, $H_B \subseteq V_B \cap F_Y = N_B$ and $M_B \cap N_B \subseteq U_B \cap V_B = F_\Phi$. Thus, (F_Y, \tilde{m}_{F_Y}) is soft T_3 -space.

4.13. Definition: A soft minimal space (F_A, \tilde{m}) is said to be soft normal if for each pair of soft \tilde{m} -closed sets G_B and H_B in F_A and such that $G_B \cap H_B = F_\Phi$, there exists soft \tilde{m} -open sets U_B and V_B such that $G_B \subseteq U_B$, $H_B \subseteq V_B$ and $U_B \cap V_B = F_\Phi$.

4.14. Theorem: A soft minimal space (F_A, \tilde{m}) is soft normal if and only if for every soft \tilde{m} -closed set G_B in F_A and soft \tilde{m} -open set U_B in F_A containing G_B , there exist soft \tilde{m} -open set V_B such that $G_B \subseteq V_B \subseteq \tilde{m}Cl(V_B) \subseteq U_B$.

Proof: Suppose (F_A, \tilde{m}) is soft normal space, let G_B be soft \tilde{m} -closed set in F_A and U_B is soft \tilde{m} -open in F_A such that $G_B \subseteq U_B$. Then $(U_B)^C$ is soft \tilde{m} -closed in F_A and $G_B \cap (U_B)^C = F_\Phi$. So there exists soft \tilde{m} -open sets V_B and K_B such that $(U_B)^C \subseteq K_B$, $G_B \subseteq V_B$ and $V_B \cap K_B = F_\Phi$, $(K_B)^C \subseteq U_B$, $V_B \subseteq (K_B)^C$. This implies that $\tilde{m}Cl(V_B) \subseteq \tilde{m}Cl(K_B)^C = (K_B)^C \subseteq U_B$. Then $G_B \subseteq V_B \subseteq \tilde{m}Cl(V_B) \subseteq U_B$.

International Journal of Innovative Research in Science, Engineering and Technology

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Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

Conversely, let G_B and H_B be soft \tilde{m} -closed sets in F_A such that $G_B \cap H_B = F_\Phi$, then $(H_B)^C$ is soft \tilde{m} -open set in F_A , and $G_B \subseteq (H_B)^C$. By hypothesis, there exist soft \tilde{m} -open set V_B such that $G_B \subseteq V_B$, $\tilde{m}Cl(V_B) \subseteq (H_B)^C$, then $H_B \subseteq [\tilde{m}Cl(V_B)]^C$. So we have $G_B \subseteq V_B$, $H_B \subseteq [\tilde{m}Cl(V_B)]^C$ and $V_B \cap [\tilde{m}Cl(V_B)]^C = F_\Phi$. Therefore, (F_A, \tilde{m}) is soft normal space.

4.15. Theorem: A soft closed subspace of a soft normal space is soft normal.

Proof: Let (F_A, \tilde{m}) be a soft normal space. Let (F_Y, \tilde{m}_{F_Y}) be soft closed subspace of (F_A, \tilde{m}) . Let M_B and N_B be two soft disjoint soft \tilde{m}_{F_Y} closed subsets in (F_Y, \tilde{m}_{F_Y}) . Then there exists soft \tilde{m} -closed sets G_B and H_B in F_A such that $M_B = G_B \cap F_Y$, $N_B = H_B \cap F_Y$. Hence by soft normality there exist soft \tilde{m} -open sets U_B and V_B such that $M_B \subseteq U_B$, $N_B \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. Since $M_B, N_B \subseteq F_Y$, then $M_B \subseteq U_B \cap F_Y$, $N_B \subseteq V_B \cap F_Y$ and $(U_B \cap F_Y) \cap (V_B \cap F_Y) = F_\Phi$, where $U_B \cap F_Y$ and $V_B \cap F_Y$ are soft \tilde{m}_{F_Y} -open sets in F_Y . Therefore, (F_Y, \tilde{m}_{F_Y}) is soft normal space.

4.16. Definition: Let (F_A, \tilde{m}) be a soft minimal space. Then (F_A, \tilde{m}) is said to be soft T_4 -space, if it is a soft normal and soft T_1 -space.

4.17. Proposition: Every soft T_4 -space is also soft T_3 -space

Proof: Let (F_A, \tilde{m}) be a soft T_4 -space, then (F_A, \tilde{m}) is soft normal as well as soft T_1 -space. To prove that the space is soft T_3 -space, it suffices to show that the space is soft regular.

Let K_B be a soft \tilde{m} -closed soft subset of F_A and let (x, u) be a soft point of F_A , such that $(x, u) \notin K_B$. Since (F_A, \tilde{m}) is soft T_1 -space. Thus $\{(x, u)\}$ is a soft \tilde{m} -closed soft subset of F_A , such that $\{(x, u)\} \cap K_B = F_\Phi$, then by soft normality, there exist soft \tilde{m} -open sets U_B and V_B in F_A such that $\{(x, u)\} \subseteq U_B$ and $K_B \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. Also $\{(x, u)\} \subseteq U_B$, then $(x, u) \in U_B$, then there exist soft \tilde{m} -open sets U_B and V_B such that $\{(x, u)\} \subseteq U_B$, $K_B \subseteq V_B$ and $U_B \cap V_B = F_\Phi$. It follows that the space (F_A, \tilde{m}) is soft regular and so soft T_3 -space.

4.18. Remark: Every soft T_4 -space is also soft T_3 -space; every soft T_3 -space is also soft T_2 -space.

4.19. Definition: A soft minimal space (F_A, \tilde{m}) is said to be completely soft normal if for each pair of soft disjoint soft \tilde{m} -closed sets U_B and $V_B \in F_A$, there exist soft \tilde{m} -open sets K_B and L_B such that $U_B \subseteq K_B$, $V_B \subseteq L_B$ and $\tilde{m}Cl(K_B) \cap \tilde{m}Cl(L_B) = F_\Phi$.

4.20. Theorem: If (F_A, \tilde{m}) is completely soft normal then it is soft normal.

Proof: Let (F_A, \tilde{m}) is completely soft normal minimal space. Let U_B and V_B are any two soft disjoint soft \tilde{m} -closed sets. Then by completely soft normality, we have there exist soft \tilde{m} -open sets K_B and L_B such that $U_B \subseteq K_B$, $V_B \subseteq L_B$ and $K_B \cap L_B = F_\Phi$.

4.21. Theorem: If (F_A, \tilde{m}) is completely soft normal then every soft subspace is soft normal.

Proof: Let (F_A, \tilde{m}) is completely soft normal and (F_Y, \tilde{m}_{F_Y}) be soft subspace. Let M_B and N_B are two soft disjoint soft \tilde{m}_{F_Y} -closed sets in (F_Y, \tilde{m}_{F_Y}) . Then $M_B = U_B \cap F_Y$ and $N_B = V_B \cap F_Y$, where U_B and V_B are soft \tilde{m} -closed sets in (F_A, \tilde{m}) . Then there exist soft \tilde{m} -open sets K_B and L_B in F_A such that $U_B \subseteq K_B$, $V_B \subseteq L_B$ and $K_B \cap L_B = F_\Phi$. This implies $U_B \cap F_Y \subseteq K_B \cap F_Y$, $V_B \cap F_Y \subseteq L_B \cap F_Y$ and $(K_B \cap F_Y) \cap (L_B \cap F_Y) = F_\Phi$. Thus, (F_Y, \tilde{m}_{F_Y}) is soft normal.

V. CONCLUSION

In this paper, we introduce the notion of separation axioms in soft minimal spaces. In particular, we study the properties of soft T_i , $i = 0, 1, 2$, soft regular and soft normal spaces. We hope that, the results in this paper will help researcher enhance and promote the further study on soft minimal to carry out general framework for their application in practical life.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Website: www.ijirset.com

Vol. 6, Issue 7, July 2017

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